Neutrino induced resonance production

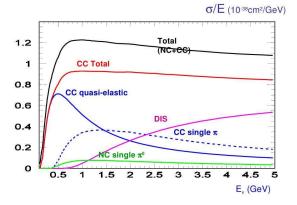
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Outline

- One-pion production as resonance production + background
- Resonance production
- Background
- Muclear effects
- 5 Topics to discuss rather than conclusion

The total cross section



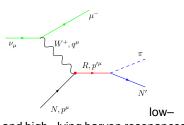
$$\sigma_{tot} = \sigma_{QE} + \sigma_{1\pi} + \sigma_{DIS}$$

- quasi-elastic (QE)
 ν_In → I⁻p
- one-pion-production $\nu_l N \to I^-(\nu_l) \pi N'$
- deep inelastic (DIS) $\nu_I N \to I^- X$

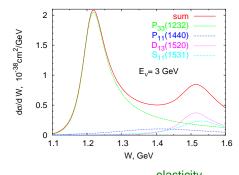
One–pion production as resonance production + background

- Resonance production (RES) peak in W (or ν) distribution $\nu_l N \to l^- R \to l^- N' \pi$ (Charged Current) $\nu_l N \to \nu_l R \to \nu_l N' \pi$ (Neutral Current)
- background smooth function of W (or ν) (Walker, 1969)
- ??? resonance-background interference

Isobar model for resonance production



and high-lying baryon resonances



			Clasifold
Risospin, spin	M_R , GeV	$\Gamma_{R(tot)}$, GeV	$\Gamma_R(R o \pi N)/\Gamma_{R(tot)}$
$P_{33}(1232)(\Delta^{++},\Delta^{+},\Delta^{0},\Delta^{-})$	1.232	0.114	0.995
$P_{11}(1440)(P_{11}^+, P_{11}^0)$	1.440	0.350(250 - 450)	0.6(0.6-0.7)
$D_{13}(1520)(D_{13}^+, D_{13}^0)$	1.520	0.125(110 - 135)	0.5(0.5-0.6)
$S_{11}(1535)(S_{11}^+, S_{11}^0)$	1.535	0.150(100 – 250)	0.4(0.35 - 0.55)

Leptonic vertex is known — independent on the resonance being produced

Theoretical model for each resonance production vertex is needed

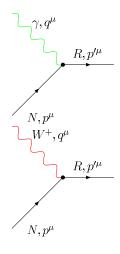
Modelling low-energy neutrino cross sections: from QE to DIS

- QE scattering $\nu_I N \to I^- N$ nucelon degrees of freedom Llewellyn-Smith formula for scattering on free nucleon, which express cross section via the vector and axial *nucleon form factors* $F_V(Q^2) = \frac{F_V(0)}{D_V}, \quad D_V = (1 + Q^2/M_V^2), \quad M_V = 0.84 \ {\rm GeV} \qquad \text{"dipole behavior"}$ $F_A(Q^2) = \frac{F_A(0)}{D_A}, \quad D_A = (1 + Q^2/M_A^2), \quad M_A = 1.05 \ {\rm GeV} \qquad \text{"axial mass} \equiv {\rm QE} \ M_A^{\rm mass} = {$
- RES: resonance production
 1) nucleon degrees of freedom: transition form factors exhibit non-dipole behaviour
 - ?? what is "one-pion axial mass": can we introduce it and how to do this?
 2) quark degrees of freedom: predicting form factors from *some* quark model
 Rein-Sehgal model Ann. Phys. 133 (1980) of resonance production is based on
 relativistic quark model (used to date in neutrino event generators)
- standard DIS formula for high W and Q^2 quark degrees of freedom difficulty: joining the resonance and DIS region, avoiding double counting

Theoretical models for resonance production on nucleon

- isobar model for P₃₃(1232) resonance (Δ), cross section in Schreiner, Von Hippel, 1973
 - the structure of the resonance-production vertex is given
 - form factors can be improved (in particular vector form factors beyond the magnetic dominance)
 - the similar model + adopting the appropriate form factors for other resonances
- phenomenological model of Dortmund group (fitting helicity amplitudes for the first 4 resonances); OL, Paschos PRD 74
- more resonances included by Giessen group
- refitting axial form factor of P₃₃(1232) (including background) Hernandez, Nieves, Valverde, PRD 76, Graczyk, Sobczyk, PRD 77
- Rein-Sehgal model (18 resonances) Ann. Phys. 133 (1980), based on the
 relativistic quark model; update for massive outgoing mesons by , K.Hagiwara et al
 (KEK), Graczyk, Sobczyk, PRD 77 difficulty: not so easy to fine-tune the model
 for a better agreement with the data
- Sato-Lee model (dynamical coupled-channel model of meson production, including background) for P₃₃(1232) resonance PRC 67

Phenomenological form factors



The electromagnetic hadronic vertex is parametrized in terms of the electromagnetic nucleon-resonance (transition) form factors $C_i^{(\rho)}(Q^2)$ and $C_i^{(n)}(Q^2)$, which in general case do not coincide for proton and neutron

The weak hadronic vertex is parametrized in terms of the weak nucleon-resonance (transition) vector $C_i^V(Q^2)$ and axial $C_i^A(Q^2)$ form factors

Several form factors must be used for each resonance:

spin-3/2: 3 vector and 4 axial

spin-1/2: 2 vector and 2 axial (at least)

X-sec is expressed via these form factors ⊕ kinematics

General situation: a lot of parameters to fit

How the form factors can be determined

- Theory predictions: no precise description even for the electroproduction data is available (see JLab, Bonn)
- So ... Theoretical ideas + phenomenology
 1st step: Weak vector form factors can be related to electromagnetic ones due to the isospin symmetry

Relations for isospin-3/2 resonances

$$C^{(p)} = C^{(n)}$$
 electromagnetic form factors are equal for protons and neutrons $C^{(p)} = C^V$ weak vector form factors are equal to electromagnetic ones

Relations for isospin-1/2 resonances

$$C^{V} = C^{(n)} - C^{(p)}$$
 weak vector form factors are related to electromagnetic ones

using the electroproduction data (more abundant and precise than neutrino data) to determine vector form factors

- 2nd step: Weak axial form factors:
 - some can be determined from theoretical idea of PCAC (Partial Conservation of Axial Current), which: 1) relates two axial form factors to each other; 2) relates one axial form factor at $Q^2=0$ to the pion–nucleon–resonance interaction vertex (which is in turn relatively well known from pion–nucleon scattering experiments) others must be derived from comparisons with neutrino experiments (fitting the form factors)

Form factors for $P_{33}(1232)$: $(J^P = \frac{3}{2}^+)$

Earlier articles in this notation: Dufner, Tsai, PR 168, 1801; Lewellyn Smith, PR 3 (1972) 261; Schreiner, von Hippel, NPB58 (1983) 333; Paschos, Sakuda, Yu, PRD 69 (2004) 014013; Singh, Athar, Ahmad, hep-ph/0507016;

The resonance field is described by a Rarita-Schwinger spinor $\psi_{\lambda}^{(R)}$.

$$\begin{split} \langle \Delta | V^{\nu} | N \rangle &= \bar{\psi}_{\lambda}^{(R)} \left[\frac{C_3^{\nu}}{m_N} (\not q g^{\lambda \nu} - q^{\lambda} \gamma^{\nu}) + \frac{C_4^{\nu}}{m_N^2} (q \cdot p g^{\lambda \nu} - q^{\lambda} p^{\nu}) \right. \\ &\left. + \frac{C_5^{\nu}}{m_N^2} (q \cdot p' g^{\lambda \nu} - q^{\lambda} p'^{\nu}) \right] \gamma_5 u_{(N)} \end{split}$$

$$\langle \Delta | A^{\nu} | N \rangle = \bar{\psi}_{\lambda} \left[\frac{\mathbf{C}_{3}^{A}}{m_{N}} (\dot{q} g^{\lambda \nu} - q^{\lambda} \gamma^{\nu}) + \frac{\mathbf{C}_{4}^{A}}{m_{N}^{2}} (q \cdot \rho g^{\lambda \nu} - q^{\lambda} \rho^{\nu}) + \mathbf{C}_{5}^{A} g^{\lambda \nu} + \frac{\mathbf{C}_{6}^{A}}{m_{N}^{2}} q^{\lambda} q^{\nu} \right] u_{(N)}$$

How the form factors can be evaluated

- past: P₃₃(1232) (Δ) investigation began more than 40 years ago
 Comparison of the phenomenological model with the electroproduction x-sec
 (1968-1971) allows to determine vector form factors in the approximation of
 magnetic dipole dominance
 - Comparison of the Adler model ($C_3^A=0$, $C_4^A=-C_5^A/4$) with the neutrinoproduction x-sec allows to determine axial form factors (also to some accuracy level)
- present:

in 2001 unambigious evidence that not only magnetic dipole amplitude contribute, but also electric $E2 \sim -2.5\%$ and scalar $S2 \sim -5\%$ quadrupoles JLab and Mainz experiments provide information not only on x-sec but also on helicity amplitudes $A_{3/2}$, $A_{1/2}$, $S_{1/2}$ (avilable for several low–lying baryon resonances), which are related to multipoles (recall magnetic dipole dominance)

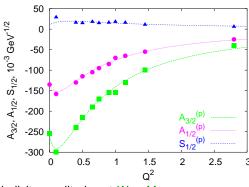
$$\sigma_T(W = M_R) = \frac{2m_N}{M_R\Gamma_R} (A_{1/2}^2 + A_{3/2}^2) \qquad \sigma_L = (W = M_R) = \frac{2m_N}{M_R\Gamma_R} \frac{Q^2}{q_Z^2} S_{1/2}^2$$

By extracting the form factor from helicity amplitudes we garanty that the accuracy of our weak vector form factors is at the same level as the accuracy of the modern electroproduction experiments (OL, Paschos, Piranishvili, PRD74) Refitting axial form factors (Hernandez, Nieves, Valverde, PRD 76; Graczyk, Sobczyk PRD 77) within the Adler model

• future: (possibly) detailed multipole analysis of both vector and axial form factors (for electroproduction 40 years of experience), going beyond the Adler model

$P_{33}(1232)$: Beyond the magnetic dominance

Magnetic dominance for low Q²: $A_{3/2} = \sqrt{3}A_{1/2}$



helicity amplitudes at $W = M_{P1232}$,

$$C_3^V = rac{2.13/D_V}{1 + Q^2/4M_V^2},$$
 $C_4^V = rac{-1.5/D_V}{\left(1 + Q^2/9M_V^2
ight)^2},$
 $C_5^V = rac{-0.4/D_V}{\left(1 + Q^2/3M_V^2
ight)^2}$
 $C_i^V = C_i^{(p)} = C_i^{(n)}$

where
$$D_V = (1 + Q^2/M_V^2)^2$$
, $M_V^2 = 0.71 \text{ GeV}^2$

For $Q^2 < 3~{\rm GeV}^2$ these form factors coincide with the "magnetic dominance" values with 4% accuracy

About vector form factors

Similar fits are also available for $P_{11}(1440)$, $D_{13}(1520)$ and $S_{11}(1535)$ resonances but the accuracy of the helicity amplitudes is not so good.

How important is going beyond the magnetic dipole dominance (MDD) for P_{33} ?

MDD:
$$A_{3/2}=\sqrt{3}A_{1/2}$$
, so $A_{3/2}$ is always bigger, $S_{1/2}=0$

Exper:
$$S_{1/2} \sim 5\% - 10\%$$
 of $A_{3/2}$ in the whole Q² region (measure up to 3 GeV)

pQCD:
$$\frac{A_{1/2}}{A_{3/2}} \rightarrow Q^2$$
 as $Q^2 \rightarrow \infty$, so $A_{1/2}$ dominates asymptotics for the form factors (Vereshkov, Volchanskiy, PRD 76) $C_3^V \sim \frac{1}{Q^6}, \qquad C_3^V \sim \frac{1}{Q^8}, \qquad C_5^V \sim \frac{1}{Q^{8+\dots}} + \text{logarithmic corrections}$

MDD:
$$C_4^V = \frac{m_N}{W} C_3^V$$
, $C_5^V = 0$ — asymptotics are not satisfied

About the so-called "one-pion axial mass"

The model of "isospin-1/2" background means NO background for $\nu p \to \Delta^{++} \to p \pi^+$ $C_5^A(Q^2)$ is determined from the data on the $d\sigma/dQ^2$ for these process the axial mass M_A was by definition taken equal to the parameter determined from the QE scattering $M_A=1.05~{\rm GeV},~D_A=\left(1+\frac{Q^2}{M_A^2}\right)^2$ (Paschos, Sakuda, Yu, PRD 69) and one additional parameter was fitted

other parametrizations of the axial form factor with several fit parameters are also available (and are equally good)

NO! "one-pion axial mass" in this approach

case (1):
$$C_5^A(Q^2) = \frac{C_5^A(0)}{D_A} \cdot \frac{1}{1 + \frac{Q^2}{3M_A^2}}$$

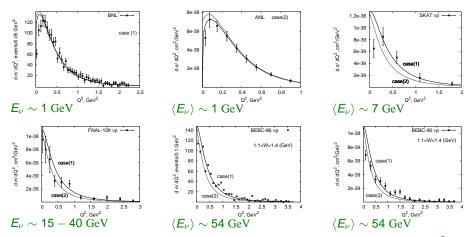
Paschos, Sakuda, Yu, PRD 69 for BNL data

case (2):
$$C_5^A(Q^2) = \frac{C_5^A(0)}{D_A} \cdot \frac{1}{1 + \frac{2Q^2}{M_A^2}}$$

Paschos, OL, PRD 71 for ANL data

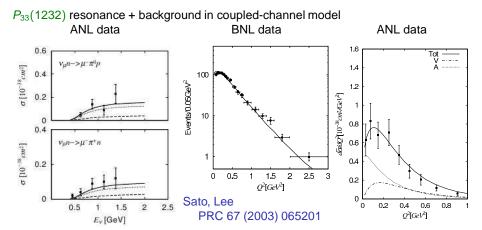
Are ANL data indeed steeper than BNL? or we just fail to describe the two experiments simultaneously within the Adler model?

$d\sigma/dQ^2$ for $\nu_{\mu} p \rightarrow \mu^- \Delta^{++} \rightarrow \mu^- p \pi^+$



Muon mass effects are noticable in this region are noticeable at low E_{ν} and low Q^2

x-sec in Sato-Lee model



Integrated cross sections are about the same as in model OL, Paschos, PRD 74 Are ANL data indeed steeper than BNL? or we just fail to describe the two experiments simultaneously within the Adler model?

- Once axial form factors are determined from $p\pi^+$ final state, we proceed with the other final states, which include the isospin-1/2 resonances and for these processes we include the "isospin-1/2" background
- The difficulty is: different experiments give rather different results, so fine tuning the models is ambigious

CC and NC reations on nucleons

Proton target

CC:
$$\nu p \rightarrow \mu^- R^{++} \rightarrow 1 p \pi^+$$
 for isospin-3/2

NC:
$$\nu p \rightarrow \nu R^+ \rightarrow \begin{cases} 1/3 & n \pi^+ \\ 2/3 & p \pi^0 \end{cases}$$
 for isospin-3/2
$$\begin{cases} 2/3 & n \pi^+ \\ 1/3 & p \pi^0 \end{cases}$$
 for isospin-1/2

Neutron target

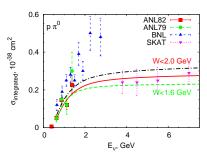
CC:
$$\nu n \rightarrow \mu^- R^+ \rightarrow \begin{cases} 1/3 & n\pi^+ \\ 2/3 & p\pi^0 \end{cases}$$
 for isospin-3/2

$$\begin{cases} \frac{2/3}{3} \frac{n\pi^+}{p\pi^0} & \text{for isospin-1/2} \end{cases}$$

NC:
$$\nu n \rightarrow \nu R^0 \rightarrow \begin{cases} \frac{1/3}{2/3} \frac{\rho \pi^-}{n \pi^0} & \text{for isospin-3/2} \\ \frac{2/3}{1/3} \frac{\rho \pi^-}{n \pi^0} & \text{for isospin-1/2} \end{cases}$$

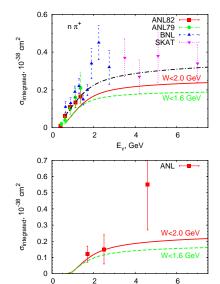
Final states:
$$\nu_{\mu} \mathbf{n} \rightarrow \mu^{-} \mathbf{R}^{+} \rightarrow \mu^{-} \mathbf{p} \pi^{0}$$
,





BNL data points are consistently higher than those of ANL and SKAT, errorbars are large for $\pi^+ n$ channel our curve is a little low than experimental points: either contributions from higher resonances or a smooth isospin-1/2 incoherent background, for example

$$\sigma_{bgr}^{\pi^+ n} = 5 \cdot 10^{-40} \left(\frac{E_{\nu}}{1 \text{ GeV}} - 0.28 \right)^{1/4} \text{ cm}^2, \\ \sigma_{bgr}^{\pi^0 p} = \frac{1}{2} \sigma_{bgr}^{\pi^+ n}$$



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E,,, GeV

x-sec in Hernandez-Nieves-Valverde model

background within sigma-model + $P_{33}(1232)$ (Δ)resonance (PRD76)

C₅^A in Hernandez–Nieves-Valverde model

The approach to calculate the x-sec: first calculate the background within the SU(2) nonlinear σ -model, then fit the form factor for the Δ -resonance Modified form factor C_5^A :

$$C_5^A(Q^2) = \frac{0.867}{(1 + \frac{Q_2}{M_A^2})^2} \times \frac{1}{1 + \frac{Q_2}{3M_A^2}}$$

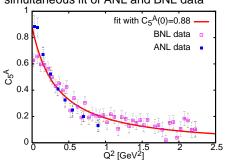
Parameter $\tilde{M}_A=0.985~{\rm GeV}$ can not be called "one–pion axial mass" because the second multiplier is kept as it was earlier, with parameter 3 (which was fitted itself in 2003 with M_A be definition equal to the QE axial mass)

- a room for improvement going beyond the Adler model and fitting C₄^A, C₃^A
- Recall, that the previous fits (by other groups) were made with the vector form factor $C_3^V(0) = 1.95$, while recent fits of helicity amplitudes give $C_3^V(0) = 2.13$ + other changes in the vector form factors; no surprise that the axial form factors are to be refitted

C₅^A within modified Rein–Sehgal model

Graczyk, Sobczyk, PRD77 combination of phenomenological (for Δ) and theoretical (Rein–Sehgal model for other resonances) arguments

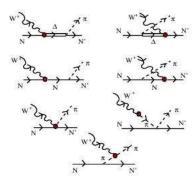
simultaneous fit of ANL and BNL data



$$C_5^A = \frac{0.88}{(1+Q^2/9.71 \text{ GeV}^2)^2} \times \frac{1}{(1+Q^2/0.35 \text{ GeV}^2)}$$

a room for improvement – going beyond the Adler model and fitting C_4^A , C_3^A

Background as a sum of Feynman diagram



picture from PRD 76

The same set of diagrams is used in the models:

- Sato-Lee (PRC 67 (2003))
- Kia-Pascalutsa-Tjon-Wright (electroproduction, PRC 70 (2004))
- Hernandez–Nieves-Valverde (PRD 76 (2007))

Iterference between $\Delta-$ and the background is considered

No direct comparison of the results of different groups is available yet

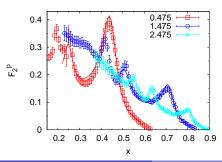
Pure phenomenological background

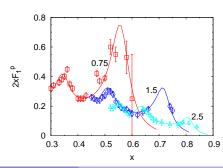
OL: fitting the JLab electroproduction data on F_2 and $2xF_1$ for different Q^2 as the first four resonances (with Dortmund form factors) + noninterfering background

$$F_2^{(p)bgr} = \frac{\nu Q^2}{Q^2 + \nu^2} \frac{a_2 (W - W_{th})^{n_2}}{(Q^2 + b_2)^3} \qquad 2x F_1^{(p)bgr} = \frac{\nu Q^2}{Q^2 + \nu^2} \frac{a_1 (W - W_{th})^{n_1}}{(Q^2 + b_1)^3}$$

 $a_1=a_2$ and $n_1=n_2$ from the requirement $F_2-2xF_1\sim 1/Q^4$ as $Q^2\to\infty$

$$W_{th} = m_N + m_\pi$$
 $a_1 = a_2 = 37.4$ $n_1 = n_2 = 0.35 + 0.22 Q^2$ $b_2 = 3.88$ $b_1 = 3.2$





Pure phenomenological background

$$F_2^{(p)bgr} = F_2^{(p)bgr}(Q^2, \nu)$$
 $2xF_1^{(p)bgr} = 2xF_1^{(p)bgr}(Q^2, \nu)$

suitable for two-, one-fold and integrated x-sec (not tested yet)

$$\frac{{\rm d}\sigma}{{\rm d}{\rm Q}^2{\rm d}\nu},\quad \frac{{\rm d}\sigma}{{\rm d}{\rm Q}^2},\qquad \frac{{\rm d}\sigma}{{\rm d}\nu},\qquad \sigma$$

- no interference
- not suitable for nuclear corrections
- what to do with the axial part?
- what to do with neutron?

Background from Giessen group

- ullet isospin-1/2 background, that is $\sigma_{bgr}^{\pi^0p}=rac{1}{2}\sigma_{bgr}^{\pi^+n}$
- idea from Rein–Sehgal: background has P11 structure without Breit–Wigner peak
- $\sigma_{bgr} = \int dW \int dQ^2 \ const \cdot {1 \over (s-m_N^2)^2} L_{\mu\nu} H_{bgr}^{\mu\nu}$
- hadronic tensor $H_{bgr}^{\mu
 u} \equiv H_{\mathrm{Q}E}^{\mu
 u}$
- const is fitted, so that the background+resonances agree with the integrated x-sec

Including nuclear effects

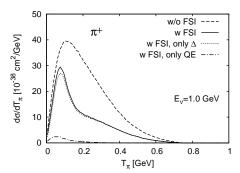
- Initial State Ineractions (ISI): we only understand interactions in the impulse approximation (neutrino hit a nucleon in the nucleus)
 - local density approximation (Giessen, Valencia–Aligarh): density distribution $\rho(r)$; medium modification of the resonance mass $M_0+Re\Sigma_\Delta(\rho)$ and width $\Gamma_0-Im\Sigma_\Delta(\rho)$
 - nuclear shell models (Gent): each nucleon is off its mass shell and is characterized by its wave function (one–particle approximation); medium modification of the resonance mass $M_0 + \overline{Re}\Sigma_\Delta$ and width $\Gamma_0 \overline{Im}\Sigma_\Delta$
 - realistic spectral function for the whole nucleus: one–particle approximation + short–range correlations (INFN, Dubna, Wroclaw, Giessen); Fermi gas model can be considered as a simplest case of these approach
- Final State Interactions (FSI): outgoing pion and nucleon propagate in nucleus
 pion absorption coefficient (average value) + charge exchange matrix
 (Dortmund)
 - relativistic optical potential, relativistic multiple scattering Glauber approximation (Gent, Madrid, Italy)
 - coupled-channel semiclassical Boltzman-Uehling-Uehlenbeck transport model (Giessen): absorption, recattering, charge exchange automatically included
 - superscaling approach (big group USA-Italy-Spain)

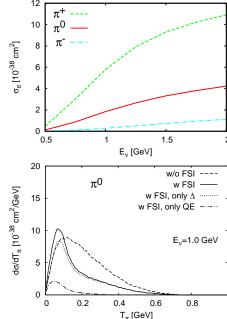
Giessen model: charged current

CC:
$$\nu_{\mu 56}$$
 Fe $\to \mu^- \pi^{+(0)} X$ (PRC 73)

FSI: absorption +rescattering +pion charge-exchange

Maximum in pion–kinetic–energy distribution is shifted to lower \mathcal{T}_{π} because the pion absorption depends on its energy (the absorption is higher in the resonance region)



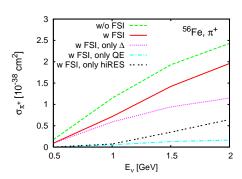


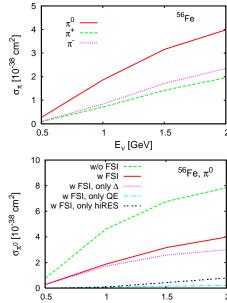
Giessen model: neutral current

NC:
$$\nu_{\mu 56}$$
 Fe $\rightarrow \nu_{\mu} \pi^{+(0)} X$ (PRC 74)

FSI: absorption +rescattering +pion charge-exchange

other distributions are available





E_ν [GeV]

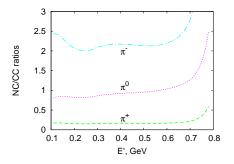
Pions of different charged in the final state (OL)

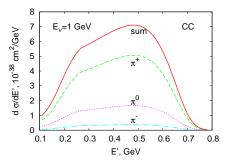
CC:
$$\nu_{\mu 12}C \rightarrow \mu^- \pi X$$

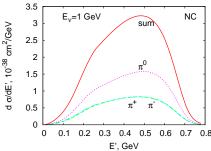
NC:
$$\nu_{\mu}_{12}C \rightarrow \nu_{\mu}\pi X$$

ISI: nuclear shell model adopted by Gent group

FSI: pion absorption coefficient and pion charge exchange matrix (in analytical form) averaged over W and Q^2 (Paschos 07041991) (reasonable for total cross section, but, probably, not for the distributions like E_{π} - or θ_{π} -)







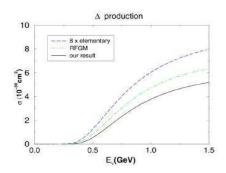
Integrated cross section with realistic specral functions

Benhar, Meloni, NPA 789

$$\nu_{\mu 16} O \rightarrow \mu^- \Delta \rightarrow \mu^- X$$

ISI: realistic spectral function (shell model + short–range NN correlations)

FSI: neglected for the inclusive final state

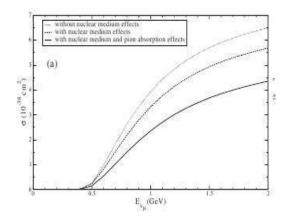


ISI: local density approximation $\rho(r)$ + medium modification of the resonance mass $M_0+Re\Sigma_{\Delta}(\rho)$ and width $\Gamma_0-Im\Sigma_{\Delta}(\rho)$

FSI: an eikonal approximation using probabilities per unit length as the basic input

Ahmad, Athar, Singh, PRD 74

$$\nu_{\mu \, 12} \textbf{C} \rightarrow \mu^- \Delta \rightarrow \mu^- \textbf{X}$$



Differential cross sections form Gent group

Praet et al

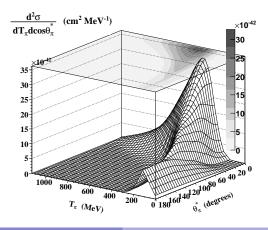
ISI: relativistic plane wave impulse approximation using realistic bound-state wave functions

FSI: relativistic Glauber model for fast ejectiles and optical potential approach for lower–energy ejectiles

$$\nu_{\mu 16} O \rightarrow \mu^- \Delta^{++} \rightarrow \mu^- p \pi^+$$

FSI in progress

$$E_{\nu}=1.3~{\rm GeV}$$



Topics to discuss rather than conclusion

- Nucleon cross section within phenomenological model
 - Vector form factors are fitted at about the same level of accuracy as electoroproduction data are available, but be aware of pQCD asymptotics at large Q²
 - Axial form factors are fitted from old neutrino cross sections, but are those experiments in agreement?
 - ▶ Is there "one-pion axial mass" or what parameters shall we fit for the axial form factors
 - What is the way to compare phenomenological results with other theory-based model (Sato-Lee, Rein-Sehgal)
- Background
 - The same Feynman diagrams are used by different groups, but how to compare results?
 - Is phenomenological background of any use?
- Nuclear corrections:
 - should be tied into other physics phenomena with photons and pions
 - accuracy versus simplicity and availability